The Division of Labor and The Impact of Intra-Industry Trade on Firm Productivity and Social Welfare

Koji Shintaku*
Graduate School of Economics, Kyoto University

Abstract

Constructing a monopolistic competition model of intra-industry trade with division of labor within the firm, this paper shows that opening trade improves firm productivity. The findings indicate that if there are fixed trade costs in trading equilibrium, opening trade improves firm productivity and raises real wage rate and makes some firms exit in the same way as Melitz (2003). In contrast to Melitz (2003), welfare doesn't necessarily rise. Whether welfare rises up or not is determined by whether the effect of rising real wage rate dominates the effect of decreasing the number of varieties or not. In particular, in a certain parameter set, opening trade improves social welfare for each country while it reduces number of varieties consumed in the world. Trade liberalization (for trade costs and number of countries) in a certain parameter set, makes the same direction changes in welfare as Melitz (2003) but a different direction changes in productivity.

Keywords: the Division of Labor within a Firm; Endogenous Firm Productivity; Monopolistic Competition; Intra-industry Trade
JEL classification numbers : F12

1 Introduction

A number of studies have formalized a model in which opening trade improves aggregate productivity through selection mechanism with firm heterogeneity (e.g. Melitz (2003), Melitz and Ottaviano (2008)). In contrast to these models, this paper develops a very simple model in which opening trade improves " firm productivity " through the division of labor within a firm.

The model is very similar to standard intra-industry trade models presented by Krugman (1980) and Melitz (2003) except for the division of labor. In particularly, we assume that number of country is $n + 1$ and each exporting firm must pay fixed trade cost for each destination country. We treat the division of labor within a firm as Chaney and Ossa (2013). Chaney and Ossa (2013) succeeds in formalizing Adam Smith's (1976) theorem, " The division of labor is limited by the extent of the market " with a model in which an expansion of labor force raises firm size (in terms of f the output or the employment) and it promotes the division of labor within the firm and improves the firm productivity ; Hereafter,

*Corresponding author. Graduate School of Economics, Kyoto University, Yoshida Honmachi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail address: shintaku.shitanku@gmail.com.

we refer to an improvement of firm productivity by firm size expansion, as *the division of labor effect*.

This paper's main results are the following. (1) Opening trade raises productivity as Melitz (2003). In contrast to Melitz (2003), it doesn't necessarily improve the welfare. (2) Trade liberalization for trade costs and number of countries makes the same direction changes in welfare as Melitz (2003) but different changes in productivity. The result for (1) implies that trade induced welfare improvement in the main model is not robust as much as in Melitz (2003). Whether welfare rises up or not is determined by whether the effect of rising real wage rate dominates the effect of decreasing the number of varieties or not. The result for (2) shows that relationship between productivity change and welfare change in this model is weaker than Melitz (2003).

We make mechanism behind the above results clear by focusing a difference between this paper's model and Chaney and Ossa (2013)'s model which this paper's model considerably depends on. Chaney and Ossa (2013) adopts a utility function driving variable markup as Krugman (1979), though this paper adopts CES-type (Dixit-Stiglitz type) utility function. Consequently, an expansion of labor force makes the competition in final good market more severe. and makes some firm exit as Melitz and Ottaviano (2008) –this selection mechanism is "pro-competition effect". This concentrates labor force to some surviving firms and raises the firm's productivity. This result implies that opening trade without trade costs raises the productivity. In contrast to Chaney and Ossa (2013), this paper's model has different selection mechanism with opening trade and trade liberalization. In this paper's setting, we can eliminate "pro-competition effect" for final goods and rather, restrict the mechanism to reallocation effect in the labor market as Melitz (2003). The model shows that opening trade doesn't raise the productivity unless exporting firms face fixed export costs. Fixed export cost is essential in the selection mechanism with the model and driving force behind the division of labor.

Research lines related to this paper are trade-induced productivity improvement (*learning by exporting*) and the division of labor within a firm. Wagner (2007) and Singh (2010) survey empirical studies about learning by exporting. They conclude that learning by exporting effect is ambiguous. The division of labor in spirit of Stigler (1951) is supported is by Levy (1984). Becker and Murphy (1992) suggests thats the division of labor isn't limited by the extent of the market and limited by coordination costs. This proposition is compatible with this paper's results. There are very few papers which analyze international trade explicitly incorporating the division of labor within a firm. Kamei (2013) is an exception. Kamei (2013) also adopts Chaney and Ossa (2013) type's the division of labor in general oligopolistic equilibrium model with variable markup rate.

The rest of the paper is constructed in the following way. Section 2 analyzes autarky equilibrium. Section 3 develops trading equilibrium and compares it with Melitz (2003). Conclusion and Appendix follow.

2 Autarky Equilibrium

In this section we analyze autarky equilibrium. We set up mainly firm's organization and then characterize the equilibrium.

2.1 Economy and Demand

There are L units of household and each household supplies one unit of labor inelastically at wage rate w . The preference of the each consumer is given by a C.E.S utility function over a continuum of goods indexed by ω :

$$U = \left[\int_{\omega \in \Omega} c(\omega)^{\rho} d\omega \right]^{1/\rho}, \; 0 < \rho < 1$$

where the measure of the set Ω represents the mass of available differentiated goods. $\sigma = 1/(1-\rho) > 1$ is the elasticity of substitution between any two goods. Price index can be obtained as

$$P = \left[\int_{\omega \in \Omega} \left(p(\omega)\right)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}. \tag{1}$$

Solving the consumer's maximization problems, a price elasticity of demand for each variety can be obtained as σ. It is constant.

2.2 Firm's organization

Each firm produces differentiated final good under the following organization structure. Many task is sequentially distributed over set $[0,2]$ in each firm as Dixit and Grossman (1982). Each firms assigns these task to t teams where $t \in R_{+}$. Since teams are symmetry, identical range of subset of the task set is assigned to each team. One unit preliminary good for a certain task set $[\underline{\omega}, \overline{\omega}]$ is produced by inputting the following units of labor

$$l([\underline{\omega}, \overline{\omega}]) = \frac{1}{2} \int_{\underline{\omega}}^{\overline{\omega}} \gamma \left| \frac{\omega_{i-1} + \omega_i}{2} - \omega \right| d\omega, \;\; \gamma > 0, \tag{2}$$

where γ is a team's burden parameter. This implies that the larger γ is , the less efficient assigning many task sets to one team is : decrease in γ raises team's performance. Figure 1 illustrates this feature for task set $[0, 4/t]$ when t is a positive integer. Integral term in (2) corresponds to area of right angled triangles in Figure 1[1].

By combining (2) for each team, one unit preliminary good for task set $[0,2]$ is produced by inputting the following units of labor [2]

$$t \left(y \int_0^{2/t} \gamma \frac{\omega}{2} d\omega \right) = t \left(y \int_0^{1/t} \gamma \omega d\omega \right) = \frac{\gamma y}{2t}.$$

One unit final good is produced inputting one unit preliminary good for task set $[0,2]$. Organizing one team requires f units of labor. Then, y units of final goods is yielded for given number of teams, t, by inputting the following units of labor

$$l(t, y) = f_d + tf + \frac{\gamma y}{2t}, \tag{3}$$

[1]For assumption of $l(\underline{\omega}, \overline{\omega})$, Chaney and Ossa (2013) adopts more general form, $l(\underline{\omega}, \overline{\omega}) = \frac{1}{2} \int_{\underline{\omega}}^{\overline{\omega}} \left(\frac{\underline{\omega} + \overline{\omega}}{2} - \omega \right)^{\alpha} d\omega$, where $\alpha >$ is a positive parameter. By assuming $l(\underline{\omega}, \overline{\omega})$ the way as (2), we can get analytical solution to a equilibrium. In Appendix A, we compare both forms in detail and show that the technology which we adopt doesn't loose generality in comparison to the one adopted by Chaney and Ossa (2013) .

[2]In right hand side of (2), by dividing the integral term by two, we can get very simple form for the units of labor.

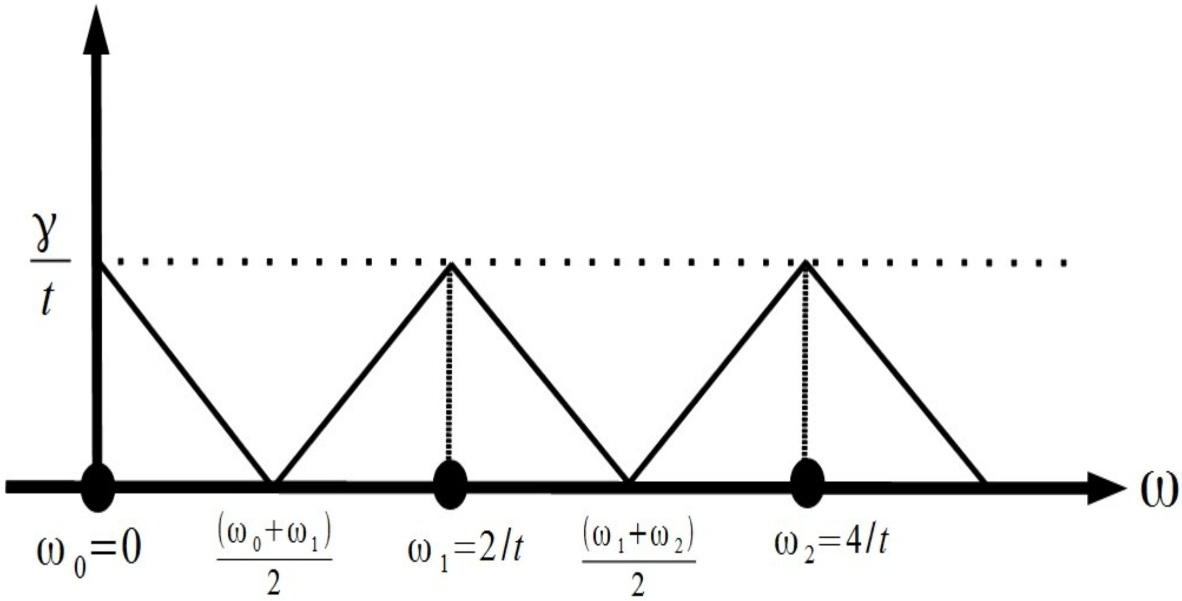

Figure 1: sequential task structure

where $f_d(>0)$ is fixed cost independent of output y and represents overhead production costs.

Each firm choose number of teams, t, so that the above labor input $l(t,y)$ is minimized. In this problem, the firm faces trade off between product improvements by increasing the number of teams and increase of organizing teams cost. The optimal number of teams t is

$$t = \left(\frac{\gamma y}{2f}\right)^{1/2}. \tag{4}$$

Combining (3) and (4) gives the following total cost function under the optimal organization

$$TC(y,w) = w(2\gamma f y)^{1/2} + w f_d. \tag{5}$$

This cost function show that the firm's technology is increasing returns to scale and further more, that marginal cost is decreasing at all levels of output.

Similarly, we can obtain the following production function

$$y = \frac{(l - f_d)^2}{2\gamma f}, \text{ for } l \geq l_d.$$

This equation shows that firm size expansion (in terms of employment l) increases marginal productivity ; $\frac{d}{dl}\frac{dy}{dl} > 0$. From (4), firm size expansion raises number of teams, which means promotion of the division of labor. Therefore, *the division of labor effect* can be confirmed. This effect is strong at low value of γ ; $\frac{d}{d\gamma}\frac{dy}{dl} < 0$.

2.3 Equilibrium allocation

We analyze firm's profit maximization problem. Each firm faces a residual demand curve with constant elasticity σ and therefore, sets pricing ; $p = \mu MC(y)$. μ is the markup rate, $\sigma/(\sigma-1)$. Using (5), this optimal pricing rule is written by PP_A schedule,

$$PP_A : \frac{p}{w} = \frac{\mu}{2}\left(\frac{2\gamma f}{y}\right)^{1/2}. \tag{6}$$

Each firm can entry and exit freely. This gives zero profit, $\pi = 0$. This is written by $p/w = AC(y)$. Using (5), this free entry condition is written by FE_A schedule,

$$FE_A : \frac{p}{w} = \left(\frac{2\gamma f}{y}\right)^{1/2} + \frac{f_d}{y}. \tag{7}$$

(6) and (7) characterize $(y, p/w)$ on equilibrium in the following way

$$y_A = \frac{{f_d}^2}{2\gamma f} B^{-2}, \tag{8}$$

$$\left(\frac{p}{w}\right)_A = B(B+1)\frac{2\gamma f}{f_d}. \tag{9}$$

where $B \equiv \mu/2 - 1$ and subscript "A" represents autarky equilibrium.

Using (8), (4) is written by

$$t_A = \frac{f_d}{2fB}. \tag{10}$$

Then, we can immediately obtain the next proposition from (8) and (9).

Proposition 1.
If $B > 0$, that is, $1 < \sigma < 2$, then, an unique internal solution $y > 0$, $p/w > 0$ exist.

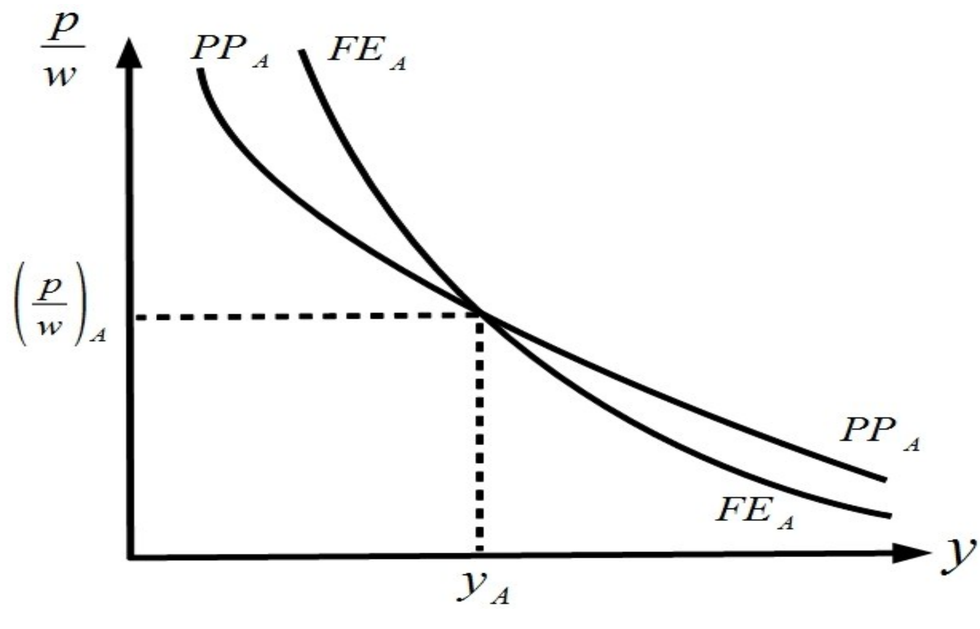

Figure 2: Autarky equilibrium in $(y, p/w)$ space.

Note that if $f_d = 0$ holds, the unique internal solution doesn't exist [3]. Hence, we must assume $f_d > 0$ to compare autarky equilibrium allocation and trading equilibrium allocation.

Figure 2 illustrates the features of autarky equilibrium. The figure1 has three intersection points between FE curve and PP curve : $(y, p/w) = (0, 0), (y_A, (p/w)_A), (\infty, \infty)$. The PP curve is cut by the FE curve only once. This ensures the unique internal solution[4].

[3]When $f_d = 0$ and $B = 0$, equilibrium output y isn't determined. When $f_d = 0$ and $B \neq 0$, equilibrium output y is zero or approaches positive infinity.

[4]The characteristic of figure 1 is supported by Appendix B.

Now, we can completely the equilibrium allocation by determining number of varieties. Labor market clearing condition, $L = Ml$, gives the following equilibrium number of varieties, M by using (3), (8), and (10), [5]

$$M_A = \frac{B}{B+1}\frac{L}{f_d}. \tag{11}$$

From (8) and (11), the next lemma is immediately obtained.

Lemma 1
An expansion of labor force doesn't raise each firm output and raise number of firms.

This results implies that *the division of labor effect* doesn't occur thorough the channel – an expansion of labor force –. [6]

An allocation in an trading equilibrium without trading costs is accord with one in integrated economy's equilibrium. Therefore, lemma 1 immediately implies the following proposition 2.

Proposition 2
Each firm productivity in an trading equilibrium without trading costs is accord with the one in autarky equilibrium.

The result of proposition 2 is identical to a result in Krugman (1980)'s section 1. This proposition leads us to consider an equilibrium with fixed export costs.

2.4 Social Welfare

We treat representative household's utility as a measure of social welfare. Under the utility maximization, indirect utility function of each household is $V = w/P$. On equilibrium, firms set identical price, p and from the definition of P, the following relation is given ;

$$V = \frac{w}{p}M^{\frac{1}{\sigma-1}}. \tag{12}$$

Note that indirect utility can be decomposed to real wage rate and number of varieties. We substitute (9) and (11) into (12) and then, obtain equilibrium social welfare, [7]

$$V_A = (2\gamma f)^{-1}L^{\frac{1}{\sigma-1}}(B+1)^{\frac{-\sigma}{\sigma-1}}B^{\frac{2-\sigma}{\sigma-1}}f_d{}^{\frac{\sigma-2}{\sigma-1}}.$$

Lemma 2
An increase in domestic fixed cost, f_d, reduces the social welfare.

proof
From assumption, $1 < \sigma < 2$,

$$\frac{dV_A}{df_d} = \frac{\sigma-2}{\sigma-1}f_d^{-1}V < 0.$$

[5] From Walras' law , budget constraint for each consumer and zero profit condition can derive equilibrium M. For simplicity, we adopt labor market clearing condition.

[6] If we doesn't impose free entry and exit condition, that is M is fixed, each firm's productivity improves. However, permitting free entry and exit, the effect of productivity improvement is outset entirely. See Shintaku (2013, b) for the details.

[7] In this equilibrium, pareto-efficient allocation is attained and then, the social welfare is maximized. See Shintaku (2013, a) for the details.

Q.E.D

An mechanism behind this result is important and explained in the following way. An increase in domestic fixed cost, f_d, raises each firm output and real wage rate through the division of labor effect and causes positive effect on the social welfare (productivity effect). On the other hand, it reduces number of varieties and causes negative effect on the social welfare (variety effect). Since the variety effect dominates the productivity effect, the social welfare decreases.[8]

We should note that the lower is the burden parameter γ, the division of labor effect is stronger ; $\frac{d}{d\gamma}\frac{d(w/p)_A}{df_d} < 0$. Furthermore, the lower is the burden parameter γ, the welfare change is smaller ; $\frac{d}{d\gamma}\frac{dV_A}{df_d} > 0$.

We see, in the next section, that fixed export cost is essential in the occurrence of *the division labor effect* under opening trade and trade liberalization.

3 Trading Equilibrium

We consider the world which consists of homogeneous $n+1$ countries. Without of loss generality, we focus on home country's allocation. Considering proposition 2's result, we assume export fixed cost as Melitz (2003).

3.1 Firm's decision

Let denote y_d as output for home market and denote y_x as the one for foreign markets. Then, we can define total out put of each firm as $y_t = y_d + ny_x$, where subscript "t" represents trading economy. Each firms faces two types of trade costs. First, firms must pay fixed cost wf_x for each export market. Second, firms must export $\tau(\tau \geq 1)$ units of products to send a foreign country one unit of products (iceberg trade cots). Then, price index is given by

$$P_T = \left[\int_{\omega\in\Omega} (p(\omega))^{1-\sigma}\, d\omega + n\int_{\omega^*\in\Omega^*} [\tau p(\omega^*)]^{1-\sigma}\, d\omega^*\right]^{1/(1-\sigma)}.$$

where, asterisk represents foreigner agent. Total cost function is given by

$$TC(y_t) = w\left[(2\gamma f y_t)^{1/2} + f_d + nf_x\right]. \tag{13}$$

Note that under the cost function, the following relation holds;

$$TC(y_t) < TC(y_d) + nTC(y_x).$$

This implies that each firm's total profit can't be decomposed to profit from home market and the one from exports markets; $\pi_t \neq \pi_d + n\pi_x$.

Let denote p_d as price for home market and denote p_x as price for export market. Mill price in export market is $p_x = \tau p_d$ from the assumption, variable trade cost τ. Then, firm profit maximization is characterized by the following optimal price setting;

$$PP : p_d = \mu MC(y_t). \tag{14}$$

[8]In standard intraindustry model with constant marginal productivity and constant markup rate, the social welfare decreases through the only variety effect.

3.2 Equilibrium allocation

The equilibrium allocation is obtained as with autarky equilibrium. From total cost function (13) and optimal price setting (14), the following equilibrium conditions are given

$$PP_T : \frac{p_d}{w} = \frac{\mu}{2}\left(\frac{2\gamma f}{y}\right)^{1/2},$$

$$FE_T : \frac{p_d}{w} = \left(\frac{2\gamma f}{y_t}\right)^{1/2} + \frac{f_d + nf_x}{y_t}.$$

These conditions give

$$y_T = \frac{(f_d + nf_x)^2}{2\gamma f}B^{-2}, \tag{15}$$

$$\left(\frac{p_d}{w}\right)_T = B(B+1)\frac{2\gamma f}{f_d + nf_x}, \tag{16}$$

$$t_T = \frac{f_d + nf_x}{2fB}, \tag{17}$$

where subscript "T" represents trading equilibrium.

Note that $y_T > y_A$, $(w/p_d)_T > (w/p_d)_A$ and $t_T > t_A$ hold. This shows that the division of labor occurs by opening trade. Why the division of labor occurs ? Remember that the only difference between aurtaky and trading equilibrium condition is the fixed cost term. Hence, increase in fixed cost term is driving force behind the division of labor.

Figure 3 illustrates the features of trading equilibrium. In figure 3, positive fixed export costs shift FE curve upward. This is the division of labor though fixed export costs.

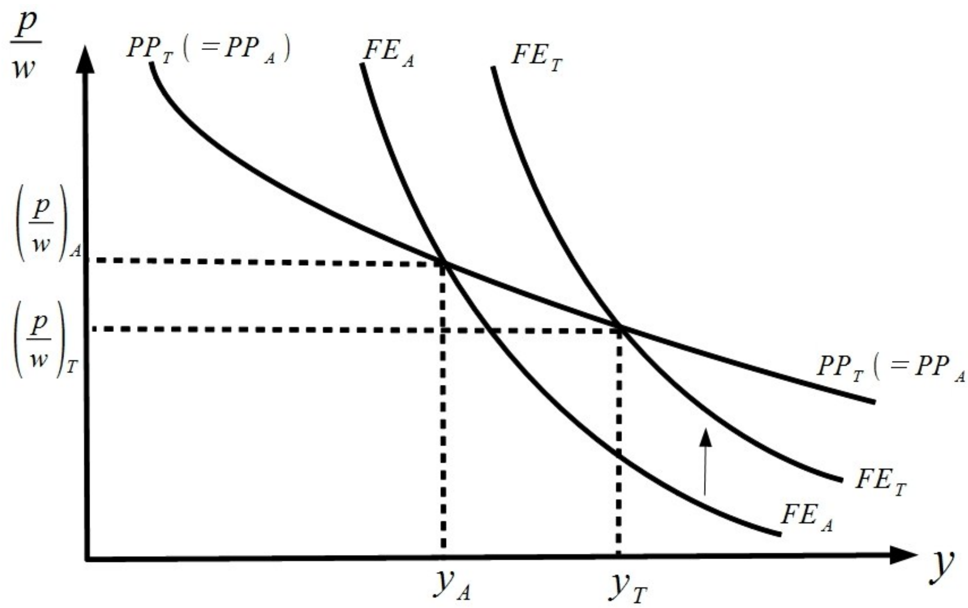

Figure 3: Tarding equilibrium in $(y, p/w)$ space.

Labor market clearing condition in open economy is given by

$$M = \frac{L}{(2\gamma f y_t)^{1/2} + f_d + nf_x} \tag{18}$$

where, M represents number of home country's firms which pay overhead production costs in the home country, f_d.

By substituting (15), y_T into (18), we cant obtain equilibrium the number of varieties;

$$M_T = \frac{B}{B+1}\frac{L}{f_d + nf_x}. \tag{19}$$

Note that $M_T < M_A$ holds. The selection mechanism is parallel to the one in Melitz (2003). Then, opening trade raises firm productivity for the surviving firms and real wage rate. This higher real wage rate makes some firms exit.

Let M_W denotes equilibrium number of world varieties which are consumed in the world. Since countries are symmetry, $M_W = (n+1)M_T$ holds. We compare M_W with M_A;

$$M_W - M_A = n(f_d - f_x)\frac{B}{B+1}\frac{L}{f_d(f_d + nf_x)}. \tag{20}$$

If $f_d \geq f_x$, then $M_W \geq M_A$ holds. The otherhand, if $f_d < f_x$, then $M_W < M_A$ holds. [9]

To sum up, we obtain the following proposition.

Lemma 3.

Opening trade raises each firm's output and team's number, real wage rate and reduces the number of home country's firms. The rest of the foreign countries is parallel to this.

Whether number of world varieties exceeds the one in autarky economy depends on whether domestic fixed cost exceeds export fixed cost.

3.3 Social welfare in each country

In trading equilibrium, indirect function is given by $V = w/P_T$. Since countries are symmetry, the following equation is obtained;

$$V = \frac{w}{p_d}M^{\frac{1}{\sigma-1}}(1 + n\tau^{1-\sigma})^{\frac{1}{\sigma-1}}. \tag{21}$$

By substituting (16) $(w/p_d)_T$ and, M_T, (19) into (21), we can obtain equilibrium social welfare;

$$V_T = (2\gamma f)^{-1}L^{\frac{1}{\sigma-1}}(B+1)^{\frac{-\sigma}{\sigma-1}}B^{\frac{2-\sigma}{\sigma-1}}(f_d + nf_x)^{\frac{\sigma-2}{\sigma-1}}(1 + n\tau^{1-\sigma})^{\frac{1}{\sigma-1}}. \tag{22}$$

This expression is written by

$$V_T = \frac{V_A}{f_d^{\frac{\sigma-2}{\sigma-1}}}(f_d + nf_x)^{\frac{\sigma-2}{\sigma-1}}(1 + n\tau^{1-\sigma})^{\frac{1}{\sigma-1}}.$$

Hence, we can immediately compare V_T with V_A. $V_T/V_A > 1$ holds under the following condition

$$1 + n\tau^{1-\sigma} > \left(1 + n\frac{f_x}{f_d}\right)^{2-\sigma}. \tag{23}$$

To sum up, we obtain the following proposition.

[9]This result just holds if we assume that technology is constant marginal productivity. See equation (C.1) in appendix C.

Proposition 3.

Opening trade changes social welfare in the following way.

(1). If $f_d \geq f_x$ holds, then, $V_T > V_A$ holds – positive productivity effect and variety effect.

(2). If $f_d < f_x$ and $1 + n\tau^{1-\sigma} \geq \left(1 + n\frac{f_x}{f_d})\right)^{2-\sigma}$ hold, then $V_T \geq V_A$ holds– positive productivity effect dominates or is equal to negative variety effect.

(3). If $1 + n\tau^{1-\sigma} < \left(1 + n\frac{f_x}{f_d})\right)^{2-\sigma}$ holds, then, $V_T < V_A$ – negative variety effect dominates positive productivity effect.

We compare the above results with the corresponding results in modified Krugman (1980) and Melitz (2003) model. We consider a modified Krugman (1980) model where assume fixed export cost and $n+1$ countries economy. In the model, if $f_x > f_d$ holds, then, number of wold varieties decreases and social welfare also decrease. The other hand, in this paper's model, (2) of proposition 3 shows that social welfare doesn't necessarily decreases under $f_x > f_d$ because of productivity effect. In Melitz (2003) model, even if umber of wold varieties decreases, opening trade raise social welfare through the productivity effect. The presence of Gains from trade is robust in Melitz (2003) model.

In the next, we compare the mechanism behind the above results with the corresponding ones in modified Krugman (1980) and Melitz (2003) model. The condition, (23) holds as long as n is high and f_x/f_d, τ is small respectively. This condition seem to be novel. In Krugman (1980) or Melitz (2003) model, all domestic firms enter export markets under the following condition[10];

$$f_d > \tau^{\sigma-1} f_x. \tag{C.2}$$

Note that n doesn't exist in this condition. The difference occurs because $TC(y_t) = TC(y_d) + nTC(y_x)$ holds in their model, while the relation doesn't hold in this paper's model.

From (22), we can implement comparative statics analysis for trade liberalization in the following way.

Proposition 4.

Trade liberalization for τ, f_x, and n has impacts on equilibrium allocation and social welfare in the following way.

(1) A decrease in variable trade cost, τ doesn't change $y_T, (w/p_d)_T, t_T, M_T$, and M_W and, raises social welfare.

(2) A decrease in fixed export cost, τ reduces $y_T, (w/p_d)_T$, and t_T and, raises M_T, M_W, and social welfare.

(3) An increase in number of trading partners, n raises $y_T, (w/p_d)_T$, t_T and social welfare and, reduces M_T. M_W is raised if $f_d > f_x$ and is reduced if $f_d < f_x$.

The mechanism behind the above results is in the following way.

A decrease in variable trade cost, τ raise effective real wage rate, $\frac{w}{p_d}(1 + n\tau^{1-\sigma})^{\frac{1}{\sigma-1}}$ and then, raises social welfare. A decrease in fixed export cost, τ has the productivity effect as lemma 2. An increase in number of trading partners, n has variety effect as lemma 3 and has productivity effect through raising nf_x as lemma 2.

Proposition 4 shows that the change in productivity by trade liberalization is very different from Melitz (2003) model while change in social welfare is parallel as the following table.

[10] This condition is relation (C.2) derived in appendix C.

This paper's model

	Aggregate productivity	number of varieties	Welfare
n	$+$	$+$ or $-$	$+$
f_x	$+$	$-$	$-$
τ	0	0	$-$

Melitz (2003)

	Aggregate productivity	number of varieties	Welfare
n	$+$	?	$+$
f_x	$-$	?	$-$
τ	$-$	?	$-$

4 Conclusion

In this paper, we got explicit the analytical solution. As shown in proposition 1, the solution exists under small parameter sets. Though the model has such disadvantage, it shows that gains from occurs through novel mechanism with very simplicity as proposition 3. Further more, the results of proposition 3 shows new view about gains form trade. The change in productivity by trade liberalization is very different from Melitz (2003). Probably, the model in this paper can be extended in various way.

5 Appendix.

5.1 Appendix A : Generality of the technology in comparison to the one adopted by Chaney and Ossa (2013)

In this section, We examine that how general the technology which adopt is in comparison to the one adopted by Chaney and Ossa (2013).

The technology we adopted is different from the one adopted by Chaney and Ossa (2013), in two points. Equation (2) in this paper corresponds to the following equation in Chaney and Ossa (2013) ;

$$l(\underline{\omega}, \overline{\omega}) = \frac{1}{2} \int_{\underline{\omega}}^{\overline{\omega}} \left(\frac{\underline{\omega} + \overline{\omega}}{2} - \omega \right)^{\alpha} d\omega. \qquad \text{(A. 2)}$$

Equation (A.2) and (2) are equal, when $\alpha = 1$ in (A.2) and $\gamma = 1$ in (2).

We examine a characteristic of parameter, α by seeing shape of $l(\underline{\omega}, \overline{\omega})$. For simplicity, we assume $\gamma = 1$ and $t = 1$. When $\alpha = 1$, integral term of right hand side in (A.2) is the area formed by "Benchmark Line" in figure 4. When $\alpha > 1$, the one is the area formed by "Curve H" in figure 4. When $0 < \alpha < 1$, the one is the area formed by "Curve L" in figure 4. Figure 4 shows that the effect of increase in α is parallel to the effect of increase in γ.

Therefore, this suggests that the technology which we adopt doesn't loose generality in comparison to the one adopted by Chaney and Ossa (2013) .

5.2 Apppendix B : Shape of PP_A curve and FE_A curve in figure 2

In this section, we examine shape of PP_A curve and FE_A curve in figure 2.

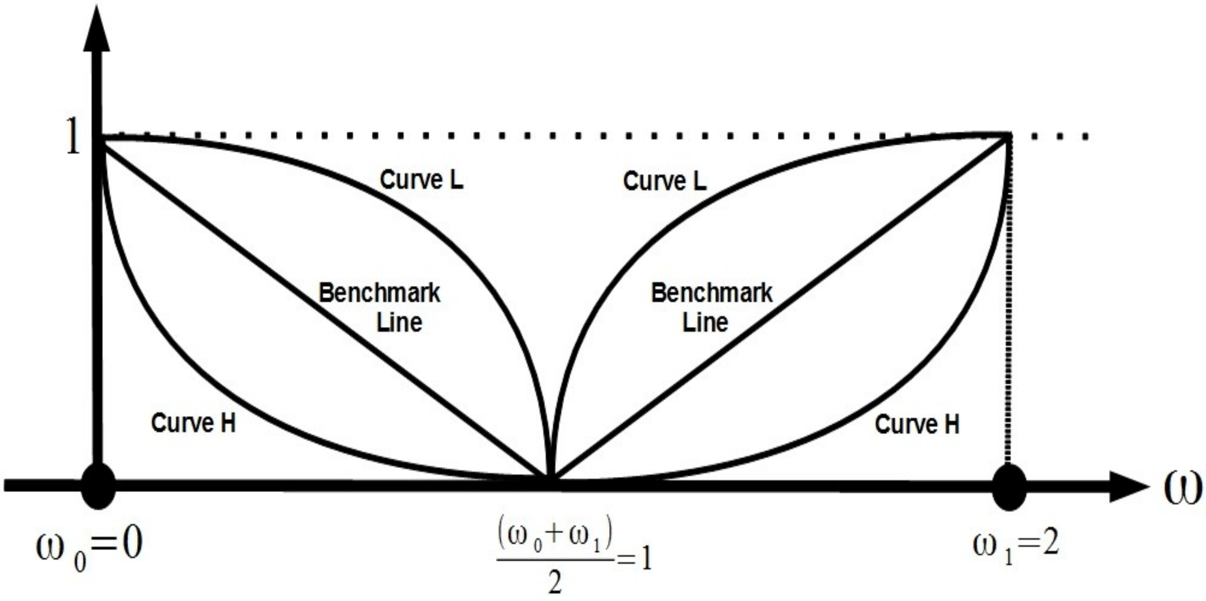

Figure 4: comparison sequential task structure

We define $Z(y)$ as difference between right hand side of (6), PP_Arelation and of (7), FE_A relation ;

$$Z(y) \equiv \frac{\mu}{2}\left(\frac{2\gamma f}{y}\right)^{1/2} - \left[\left(\frac{2\gamma f}{y}\right)^{1/2} + \frac{f_d}{y}\right] = B(2\gamma f)^{1/2}y^{-1/2} - f_d y^{-1}.$$

Certainly, $Z(y_A) = 0$ holds.

The derivative of function $Z(y)$ is given by

$$Z'(y) = -2^{-1}B(2\gamma f)^{1/2}y^{-3/2} + f_d y^{-2}.$$

When $y = y_A^*$, $Z'(y_A^*) = 0$ holds, where y_A^* is given by

$$y_A^* = 2\frac{f_d}{B^2\gamma f} = 4\frac{f_d}{B^2 2\gamma f} = 4y_A.$$

From $B > 0$, when $y < 4y_A$, $Z'(y) > 0$ holds and when $y > 4y_A$, $Z'(y) < 0$ holds. Furthermore, for the second order derivative of function $Z(y)$, $Z''(64y_A/9) = 0$ holds.

The limits of function $Z(y)$ are given by

$$\lim_{y\to\infty} Z(y) = 0,$$

$$\lim_{y\to 0} Z(y) = -\infty.$$

The above relations are proved in the following way.

Proof.

$$\lim_{y\to\infty} Z(y) = \lim_{y\to\infty} \frac{B(2\gamma f)^{1/2}y^{1/2} - f_d}{y} = \frac{0 - f_d}{\infty} \to 0,$$

$$\lim_{y\to 0} Z(y) = \lim_{y\to 0} \frac{B(2\gamma f)^{1/2}y^{1/2} - f_d}{y} = \frac{-f_d}{0} \to -\infty.$$

Q.E.D

According to the above results, the shape of $Z(y)$ is the one as shown in Figure 5.

Figure 5 is consistent to figure 2 and then, figure 2 is supported.

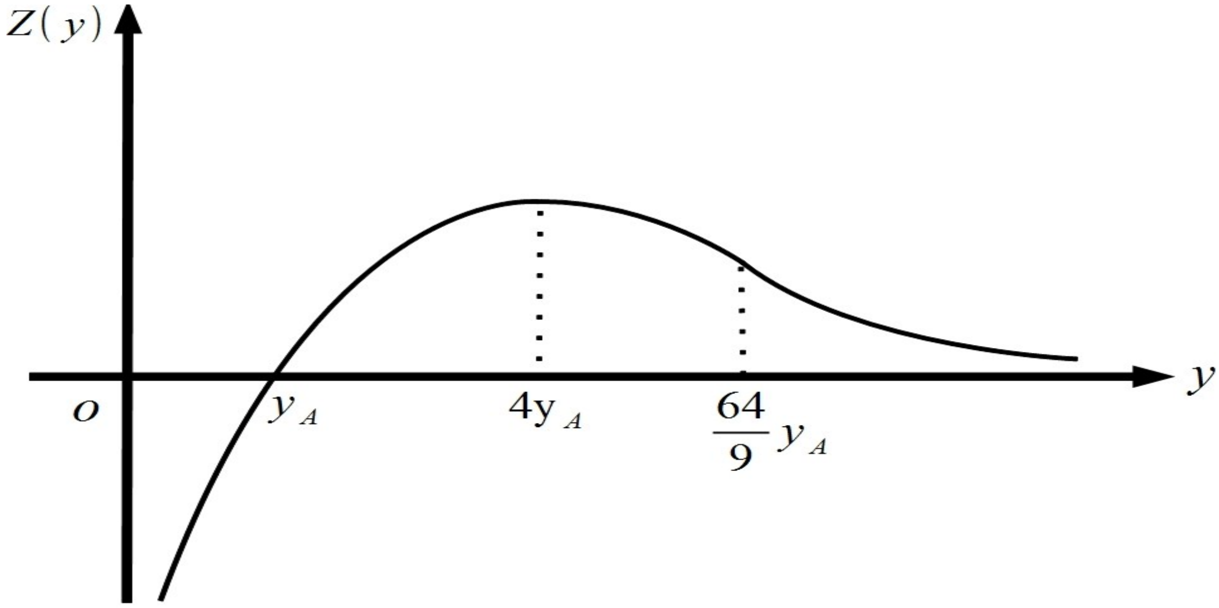

Figure 5: the shape of Z(y)

5.3 Appendix C : The impact of trade with non-division of labor

In this section, we derives equilibrium number of variety and social welfare under technology with constant marginal productivity.

We assume thta firms produce the product with constant marginal productivity, ψ. Then, total cost function in autarky and open economy are given by, respectively,

$$TC(y) = w\left(\frac{y}{\psi} + f_d\right),$$

$$TC(y) = w\left(\frac{y}{\psi} + f_d + nf_x\right).$$

Under these cost functions, we examine the change in number of varieties by opening trade. Equilibrium number of varieties in autarky and open economy are given by, respectively,

$$M_A = \frac{2B+1}{2(B+1)}\frac{L}{f_d},$$

$$M_W = (n+1)\frac{2B+1}{2(B+1)}\frac{L}{(f_d + nf_x)}.$$

The change in number of varieties is

$$M_W - M_A = n(f_d - f_x)\frac{2B+1}{2(B+1)}\frac{L}{f_d(f_d + nf_x)}. \tag{C.1}$$

(C.1) shows that lemme 3 holds under these technology.

In the next, we examine the change in social welfare by opening trade. Equilibrium social welfare in autarky and open economy are given by, respectively,

$$V_A = L^{\frac{1}{\sigma-1}}\sigma^{-\mu}(\sigma-1)\psi f_d^{\frac{-1}{\sigma-1}},$$

$$V_T = V_A f_d^{\frac{1}{\sigma-1}} (f_d + nf_x)^{\frac{-1}{\sigma-1}} (1 + n\tau^{1-\sigma})^{\frac{1}{\sigma-1}}.$$

From these equations, $V_T/V_A > 1$ holds under the following condition

$$f_d^{\frac{1}{\sigma-1}} (f_d + nf_x)^{\frac{-1}{\sigma-1}} (1 + n\tau^{1-\sigma})^{\frac{1}{\sigma-1}} > 1.$$

This relation is written by

$$f_d > \tau^{\sigma-1} f_x. \tag{C.2}$$

(C.2) means that if domestic fixed cost exceeds combination of variable and fixed trade costs, gains from trade exists.

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